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Stabilized spatiotemporal waves in a convectively unstable open flow system: coupled diode resonators

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With various methods we demonstrate the establishment of stable, spatially extended wave forms underlying a spatiotemporally chaotic state in open flow systems consisting of coupled oscillators. Results are obtained for an experimental system consisting of unidirectionally coupled diode resonator circuits as well as for the coupled map lattice, a numerical model made up of coupled logistic maps. Both systems exhibit convective instability and high-dimensional, complex spatiotemporal behavior. In each system spatial wave forms are stabilized by fixing appropriate temporal periods at the first oscillator. The other elements assume the periodicity of the first, yet exhibit spatially varying amplitudes which have an associated wavelength and are in general spatially quasiperiodic.

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The recent surge in the development and application of control techniques for temporally chaotic systems of varying complexity has naturally sparked an interest in the control of spatially extended, or spatiotemporal systems [1–6]. While "spatiotemporal" covers a diverse class of numerical and physical systems, here we focus on one-dimensional systems that possess a preferred direction to propagate information out of the system, so-called open flows [7]. Fluid flow in a pipe and channel flow are typically cited prototypes of open flow systems in which a transition from a coherent structure (laminar flow) to fully developed turbulence further downstream is observed within certain parameter ranges. In these instances, open flow systems become *convectively unstable*: microscopic fluctuations are amplified exponentially while being convected through the system. The simplest and computationally most feasible model of an open flow system is the one-dimensional lattice of asymmetrically coupled logistic maps, as extensively studied by Kaneko [8]. Being conceptually uncomplicated (as compared to the equations of motion for, e.g., a fluid flow), it nevertheless displays highly complex and rich patterns. With the first site fixed on the unstable fixed point of a single logistic map, one observes spatial period doubling (a doubling of the temporal periodicity as the spatial index is increased) leading to chaos at higher sites. This phenomenon was shown by Liu and Gollub to be involved in the transition from periodic waves to spatiotemporal chaos in one-dimensional film flow [9].

Coupled map systems have provided a convenient testing ground for pioneering work in the area of controlling spatiotemporal chaos. Auerbach [3] has demonstrated the stabilizing of convectively unstable periodic states in an open flow model using modified chaos control feedback techniques. By employing feedback based on local information to sites distributed periodically in space, spatial period doubling was suppressed and coherence maintained throughout the length of the lattice. The resulting states are temporally periodic and spatially uniform. Meanwhile, Gang and Zhilin [2] have stabilized very different states that are periodic in time *and* space in an otherwise chaotic coupled map lattice. In this case the system possessed symmetric coupling and periodic boundary conditions, hence no preferred direction of propagation. Similarly, methods have been successful in controlling chaos in spatially continuous media described by nonlinear, one-dimensional partial differential equations.

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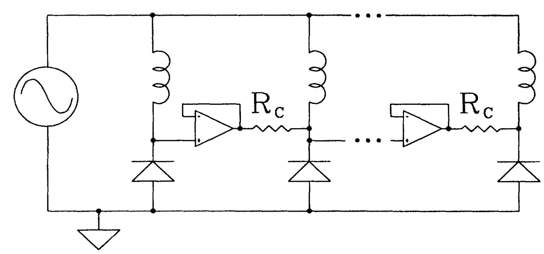

FIG. 1. Open flow circuit consisting of 32 diode resonators coupled unidirectionally to the right. The strength of the coupling is dependent on the coupling resistor R_C.

Gang and Kaifen [1] and Aranson *et al.* [5] have shown that a single control point in space is sufficient to stabilize extended systems of limited size. In this paper we demonstrate single-point control of spatially varying states in an open flow lattice of arbitrary size.

Recently, Willeboordse and Kaneko [10] found that under certain conditions in an open flow coupled map lattice, spatial period-doubling bifurcations lead to states which are temporally periodic and spatially quasiperiodic. Once established, these structures presumably persist out to infinity in the spatial direction, indicating their stable nature and apparent immunity to noise amplification. However, the spatial region preceding the stable wave forms remains convectively unstable, thus highly sensitive to *any* level of noise. Consequently, the addition of the smallest noise levels (i.e., in the last digit) destroys all stable structure downstream through macroscopic fluctuations, resulting in complex, turbulent behavior. Because of the extreme sensitivity to noise, we do not expect that these stable states will be found in an experiment by the method in [10], that is, fixing the first site to a constant value. We report that by controlling the first site into properly chosen periodic orbits, states similar to those reported above become stable throughout the length of the lattice. We demonstrate this phenomenon in an experimental system of coupled electronic circuits and in a system of coupled logistic maps.

Our experimental system consists of 32 coupled diode resonator circuits [11] driven by a sinusoidal source at a frequency of 70 kHz. The individual circuits are comprised of the series combination of a 30 mH inductance and a General Instruments 852 silicon diode. Individually, the circuits follow the period-doubling route to chaos as the drive voltage is increased and form nearly one-dimensional first return maps by mapping the peak current through the diode. The diodes were matched based on their bifurcation sequences such that all go chaotic at nearly the same drive voltage. Placing a buffer and resistor between neighboring diodes, as shown in Fig. 1, provides a one-way coupling proportional to the difference $(V_d^i - V_d^{i-1})$, where V_d is the voltage across the diodes and i represents the spatial index. The coupling strength is determined by the coupling resistor, R_C, chosen to be 18.2 kΩ. This choice results in spatial features with appropriate scale in relation to the size of our system. The boundary conditions are open so that the first circuit is independent of the rest.

Throughout the experiments, the system is driven well within the chaotic regime of the individual circuits (drive voltage = 2.7 V_{rms}). The resulting state is illustrated in Fig. 2(a). Snapshots of the system are taken by simultaneously sampling the current at all sites. In the figure, 11 snapshots taken at arbitrary times are plotted and indicated by the lines connecting the simultaneous data points. The coupling between sites has a synchronizing effect which slaves the first few sites to the dynamics of the first, as indicated by the flat regions at low site numbers. The convectively unstable nature of the system works against the synchronizing effect and eventually produces complex, high-dimensional dynamics downstream. Keeping the drive voltage the same, if we fix the first diode resonator to the unstable period-1 fixed point [12], the following few sites assume the period-1 orbit as in Fig. 2(b). The period-1 solution gives way to a period-2 orbit for a number of sites, and eventually all periodic behavior is destroyed for high site numbers.

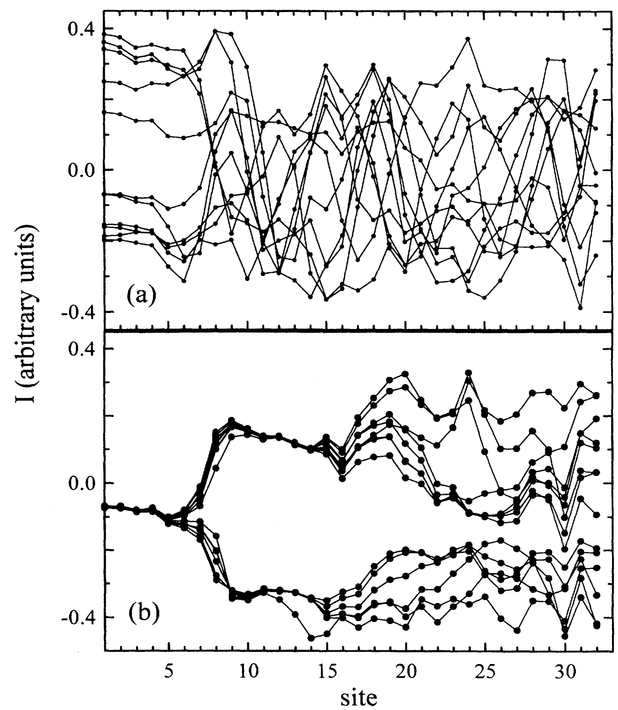

FIG. 2. The dynamics of the diode resonator open flow system. In both plots several snapshots of the system are taken, the lines connect the current peaks, I, sampled simultaneously. (a) The first site is chaotic, the next few sites follow nearly the same trajectory, and eventually more complex behavior develops after about 10 sites. (b) The first site is controlled in a period-1 orbit leading to a spatial period-doubling bifurcation and eventual chaos downstream.

Although spatially coherent or laminar states are convectively unstable and give way to spatial period doubling and turbulence, we have found that controlling certain periodic orbits at site 1 results in stable spatial wave forms which extend throughout the length of the lattice of diode resonators. An example is given in Fig. 3 in which the first site has been stabilized into a period-5 orbit. Typically, a flat, spatially coherent region is formed by the first few sites before the spatial oscillation sets in. This region, which is barely 3 sites in the figure, is sensitive to the control parameters as well as the coupling resistance. The spatial structure of the period-5 state is characterized by a wavelength of approxi-

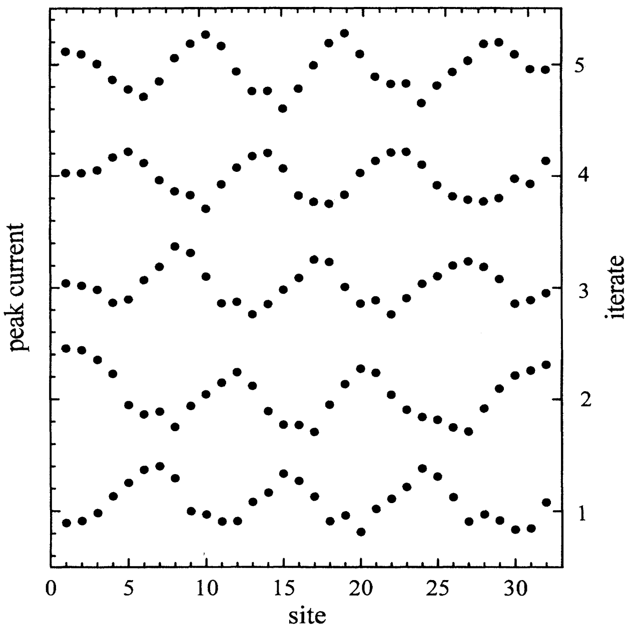

FIG. 3. The spatial waves associated with a controlled period 5 at site 1 in the diode resonator system. After a short spatial transient, this spatiotemporal state is characterized by a wavelength of approximately 8.3 sites and a temporal winding number of 2/5. Every site is in a period-5 orbit, so the subsequent iterates plotted repeat every five cycles.

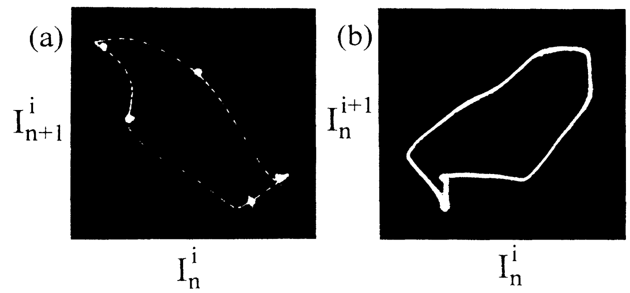

FIG. 4. The dynamics of the closed-loop system of diode resonators. (a) Double exposure of a single return map at any site i in the loop. The distorted ringlike map indicates the quasiperiodic nature of the system, and the five bright dots show a period-5 locked state at a nearby drive voltage. (b) Spatial return map, a mapping of neighboring sites at equal times. The map indicates the phase difference between sites and determines the spatial structure of the wave forms. The time exposure taken in the quasiperiodic regime shows multiple n values, making the map appear continuous.

mately 8.3 sites, so that there is an average phase shift of about 43° between sites at a given time. Besides the period-5 state, we have stabilized waves with temporal periods including 7, 8, 9, 11, 13, 14, 18, and 20 in the same manner.

By employing periodic boundary conditions in our system, i.e., coupling the last site to the first, we observe quasiperiodic behavior in the individual diode resonators and an integral number of spatial wavelengths in the loop. For a drive voltage of 2.30 V_{rms}, the return map at any given site i and cycle number n, I^i_{n+1} vs I^i_n, is a distorted ringlike attractor as shown in the double exposure photograph of Fig. 4(a). The bright dots indicate a period-5 mode-locked state at a slightly different drive voltage. In this state the map reveals that the temporal winding number of the individual elements of the system is 2/5, i.e., the attractor viewed as a rough circle is traversed twice per five drive cycles. The spatial behavior is characterized by Fig. 4(b) in a plot of I^{i+1}_n vs I^i_n simultaneously sampled neighboring currents. This so-called spatial return map shows the relative phase difference between neighboring sites. In the quasiperiodic regime, this time-exposure photo (many n values) produces a continuous map, which appears identical for any two neighboring sites in the lattice. At different drive amplitudes the maps reveal very rich, high-dimensional dynamics as well as numerous windows of stable mode-locked states with periods from five to hundreds of drive cycles. Spatially the locked states form wave patterns which have an integral number of wavelengths in the length of the lattice. Changing the lattice size reveals similar waves of different temporal period and spatial wavelength. For the coupling resistance used, we observe temporal winding numbers between 0.38 and 0.44 and wavelengths from 7 to 16 sites. The tendency of the system to favor a winding number near 0.4 is consistent with some low-period orbits being unobserved as solutions of the system (e.g., periods 2, 3, 4, and 6). By applying periodic boundary conditions we have established the existence of stable spatiotemporal waves in the system, some of which we observe in the open flow system by controlling the proper orbit at the initial site.

Presumably, to stabilize the open flow system, the requirement is to mimic the dynamics of a single element of the closed-loop locked states at the first site by establishing the desired temporal periodicity. We do this in one of two ways: (i) by applying one of the recently developed chaos control techniques to site 1 as described above, or (ii) by simulating a "site 0" with an arbitrary wave form generator. In the latter case we digitally store the voltage trace from a single element of the closed-loop system in a mode-locked state. Playing back the stored wave form, we apply the output of the wave form generator to the first diode through a resistor of size R_C. The otherwise chaotic open flow system then becomes stabilized into the previously observed mode-locked state. Typically, in method (i), once the temporal periodicity of the first diode resonator is established, the system downstream assumes a state observed in the closed loop. The first few sites serve as transient states which connect the dynamics of the one-dimensional temporal return map of the first site to the significantly different, circular maps of the closed loop system. In the second method, no transient is observed since the dynamics induced at site 1 match the locked state downstream.

Finally, by coupling a lower frequency sine wave to site 1, we have found that it is sufficient to stabilize the system by introducing an appropriate frequency. The method is similar to method (ii) in that the sine wave simulates a diode resonator voltage in a periodic orbit whenever the ratio of the 70 kHz drive to the wave generator frequency is rational. Some of the extremely high-period (>100) locked states observed in the closed loop can be reproduced with this method. For example, in the closed-loop system we observe a locked state of period 28 with winding number 11/28 and a spatial wavelength of 16 sites. Returning to the open flow system, we inject into the first site a sine wave of frequency equal to the product of the winding number and 70 kHz (27.5 kHz). All sites in the lattice assume a period-28 orbit, and again a few sites connect the sine wave at site 0 to the stable spatiotemporal state downstream.

To numerically verify our experimental results, we represented each diode resonator by a logistic map—motivated by the nearly one-dimensional nature of the return map of the diode resonator. The one-dimensional lattice of unidirectionally coupled logistic maps

$$x_{n+1}^{i}=f(x_n^i)+\epsilon[f(x_n^{i-1})-f(x_n^i)] \quad (1)$$

($i\in[1,N]$, $n\equiv$ time, $f(x)=rx(1-x)$, $x\in(0,1)$) was taken as a model for the diode resonator chain. With the system size N being chosen up to several thousand, the extreme long range stability of the waves could be verified. For the open flow case the first site was controlled into various high-period orbits. The resulting spatial wave forms are insensitive towards injection of random perturbations up to 0.05 delivered to a single element, and match the experimental results quite well. Figure 5 shows several temporal return maps of successive sites, locked into a period-5 orbit [notice the resemblance with Fig. 4(a)]. The circular shaped attractor is formed by overplotting the return maps of all sites. The line connecting simultaneous amplitudes illustrates the phase advance from site to site, picturing the spatial wave. The spatial return map is also included in the figure to illustrate the spatial quasiperiodic behavior. The almost identical orbit can be captured when applying periodic boundary conditions. Fine adjustment of the parameters then causes switching between mode-locked states, i.e., each site being periodic, and drifting waves. In the latter case the return map of a single site fills out the circlelike attractor, indicating temporal quasiperiodicity. For the locked period-5 orbit the winding number is again 2/5 and the wavelength close to six sites. Additionally, we found that exciting the first site with an appropriate sine wave stabilizes the entire coupled map system, reproducing the experimental results.

Summarizing, we have stabilized different spatial wave patterns in a convectively unstable system, both in experiment and numerical simulations. A single controller was employed at the first site of the diode resonator chain in order to stabilize the 32-element system. With a number of stabilized orbits at site 1, we observed a short coherence length followed by the abrupt formation of stable spatial wave forms throughout the lattice. Nonfeedback methods were also used to provide proper conditions at the first site resulting in stable spatiotemporal states. Results from the coupled logistic map system parallel the observations of the experiment quite well.

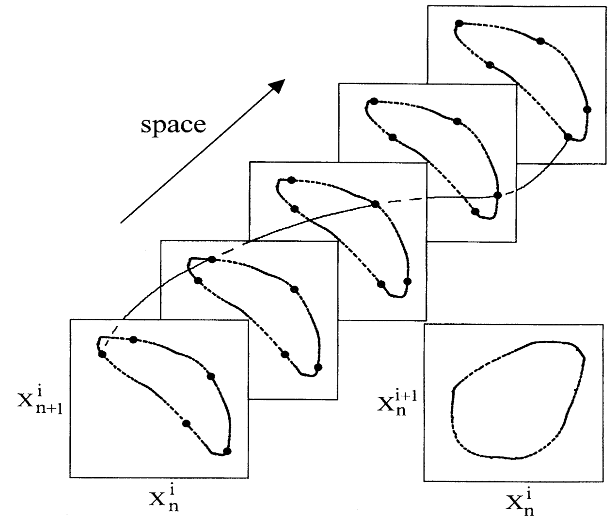

FIG. 5. The stabilized period-5 wave in the coupled map lattice. The periodic return maps for sites 260–264 are pictured as the large dots plotted over the map formed by overplotting all sites. The line connects a set of simultaneous points indicating the spatial wave form. The return maps have winding number 2/5 and bear a strong resemblance to the map of Fig. 4(a). Also pictured is the spatial return map, generated by plotting neighboring sites at a single point in time at all sites, similar to Fig. 4(b). Here $\epsilon=0.547$ and $r=3.785$.

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